SOCIETY OF ACTUARIES

AMERICAN SOCIETY OF PENSION ACTUARIES

JOINT BOARD FOR THE ENROLLMENT OF ACTUARIES

May 2000 Course EA-1, Segment 1

JOINT BOARD EXAMINATION

This is the May 2000 examination which has been released to the public by the administering organizations.

AMERICAN SOCIETY OF PENSION ACTUARIES
JOINT BOARD FOR THE ENROLLMENT OF ACTUARIES
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Mathematics of Compound Interest and
Mathematics of Life Contingencies

EA-1, Segment A

(141)

Date: Tuesday, May 16, 2000
Time: 8:30 a.m. – 11:00 a.m.

INSTRUCTIONS TO CANDIDATES

1. Write your candidate number here _____. Your name must not appear.

2. Do not break the seal of this book until the supervisor tells you to do so.

3. Special conditions generally applicable to all questions on this examination are found at the front of this book.

4. On this examination the symbol "a" will be used to represent an annuity. On this examination the symbol "l_x" will be used to represent the number of lives at age x.

5. This examination consists of 25 multiple-choice questions.

6. Each question has equal weight. Your score will be based on the number of questions which you answer correctly. No credit will be given for omitted answers and no credit will be lost for wrong answers; hence, you should answer all questions even those for which you have to guess.

7. A separate answer sheet is inside the front cover of this book. During the time allotted for this examination, record all your answers on side 2 of the answer sheet. NO ADDITIONAL TIME WILL BE ALLOWED FOR THIS PURPOSE. No credit will be given for anything indicated in the examination book but not transferred to the answer sheet. Failure to stop writing or coding your answer sheet after time is called will result in the disqualification of your answer sheet or further disciplinary action.

8. Five answer choices are given with each question, each answer choice being identified by a key letter (A to E). For each question, blacken the oval on the answer sheet which corresponds to the key letter of the answer choice that you select.

9. Use a soft-lead pencil to mark the answer sheet. To facilitate correct mechanical scoring, be sure that, for each question, your pencil mark is dark and completely fills only the intended oval. Make no stray marks on the answer sheet. If you have to erase, do so completely.

10. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.

11. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

12. Clearly indicated answer choices in the test book can be an aid in grading examinations in the unlikely event of a lost answer sheet.

13. Use the blank portions of each page for your scratch work. Extra blank pages are provided at the back of the examination book.

14. When the supervisor tells you to do so, break the seal on the book and remove the answer sheet.

    On side 1 of the answer sheet, space is provided to write and to code candidate information. Complete Blocks A through G as follows:

    (a) in Block A, print your name and the name of this test center;

    (b) in Block B, print your last name, first name and middle initial and code your name by blackening the ovals (one in each column) corresponding to the letters of your name; for each empty box, blacken the small rectangle immediately above the "A" oval;

    (c) write your candidate number in Block C (as it appears on your ticket of admission for this examination) and write the number of this test center in Block D (the supervisor will supply the number);

    (d) code your candidate number and center number by blackening the five ovals (one in each column) corresponding to the five digits of your candidate number and the three ovals (one in each column) corresponding to the three digits of the test center number, respectively. Please be sure that your candidate number and the test center number are coded correctly;

    (e) in Block E, code the examination that you are taking by blackening the oval to the left of "Exam 141 (EA1 Segment A);"

    (f) in Block F, blacken the appropriate oval to indicate whether you are using a calculator; and

    (g) in Block G, sign your name and write today's date. If the answer sheet is not signed, it will not be graded.

    On side 2 of your answer sheet, space is provided at the top for the number of this examination book. Enter the examination book number, from the upper right-hand corner of this examination book, in the four boxes at the top of side 2 marked "BOOKLET NUMBER."

15. After the examination, the supervisor will collect this book and the answer sheet separately. DO NOT ENCLOSE THE ANSWER SHEET IN THE BOOK. All books and answer sheets must be returned. THE QUESTIONS ARE CONFIDENTIAL AND MAY NOT BE TAKEN FROM THE EXAMINATION ROOM.

Conditions Generally Applicable to
All EA-1 Segment A Examination Questions

The following conditions should be considered a part of the data for each question, unless otherwise stated or implied.

(1) The normal retirement age is 65.

(2) Retirement pensions commence at normal retirement age and are paid monthly for life at the beginning of each month.

(3) There are no preretirement death or disability benefits.

(4) Actuarial equivalence is based on the mortality table and interest rate assumed for funding purposes.

(5) Interest rates which are compounded more frequently than annually are expressed as nominal rates.

(6) Where multiple lives are involved, future lifetimes are assumed to be independent of each other.

(7) The term "gross single premium" is equivalent to "contract single premium"; the term "net single premium" is equivalent to "benefit single premium"; the term "gross annual premium" is equivalent to "contract annual premium"; the term "net annual premium" is equivalent to "benefit annual premium".

(8) There are no policy loans in effect.

(9) For a bond, the face amount and the redemption value are the same.

(10) Interest rate equals yield rate.

Data for Question 1

Purchase date of a perpetuity-due: 1/1/2000.

Level payment amount: $100.

Frequency of payments: Annual.

Cost of perpetuity: $1,100.

Interest rate for perpetuity: i%, compounded annually.

Immediately following the payment on 1/1/2014, the remaining future payments are sold at a yield rate of i%. The proceeds are used to purchase an annuity certain as follows:

Term of annuity: 10 years.

First payment of annuity: 1/1/2018.

Frequency of annuity payments: Semi-annual on January 1 and July 1.

Interest rate for annuity: ½ i% compounded annually.

Question 1

In what range is the semi-annual annuity payment?

[A] Less than $75

[B] $75 but less than $77

[C] $77 but less than $79

[D] $79 but less than $81

[E] $81 or more

Data for Question 2

Smith's age at inception of mortgage: 40.

Mortgage inception date: 1/1/2000.

Amount of mortgage: $100,000.

Term of mortgage: 9 years.

Timing of payments: Annually at the end of each year.

On 1/1/2000 Smith purchases an insurance policy which will make the remaining mortgage payments in the event of his death prior to the last payment.

Interest rate: 4% per year, compounded annually.

Selected annuity factor: $\ddot{a}_{40:\overline{10}|} = 7.6923$

Question 2

In what range is the present value of Smith's death benefit on 1/1/2000?

[A] Less than $6,750

[B] $6,750 but less than $7,750

[C] $7,750 but less than $8,750

[D] $8,750 but less than $9,750

[E] $9,750 or more

Data for Question 3

Smith, age 70, considers the purchase of a single premium $10,000 whole life policy with death benefits payable at the end of the year. The net single premium is $7,000.

He decides to delay the purchase until age 73 and invests the $7,000 for the three year period.

At age 73, Smith's fund is exactly equal to the age 73 net single premium for the insurance.

Selected values:

x	l_x
70	100
71	96
72	90
73	80

Interest rate for the insurance: 6% per year, compounded annually.

Yield rate for the investment: X% per year, compounded annually.

Question 3

In what range is X%?

[A] Less than 1.9%

[B] 1.9% but less than 2.4%

[C] 2.4% but less than 2.9%

[D] 2.9% but less than 3.4%

[E] 3.4% or more

Data for Question 4

Smith, Brown, and Green are all age 81.

$e_{80} = 9.8694$

$e_{81} = 9.3315$

${}_3p_{80} = 0.8642$

Question 4

In what range is the probability that at least two of the three will die prior to attaining age 83?

[A] Less than 0.0230

[B] 0.0230 but less than 0.0240

[C] 0.0240 but less than 0.0250

[D] 0.0250 but less than 0.0260

[E] 0.0260 or more

Data for Question 5

Twenty $5,000 bonds, each with the same purchase price, are purchased on 1/1/2000. They are to be redeemed one each year starting 1/1/2015.

Coupon rate: 4% per year, payable semiannually.

Yield rate for entire series: 5% per year, compounded semiannually.

Question 5

What percentage of the bonds has a yield rate higher than 5% per year, compounded semiannually?

[A] 40%

[B] 45%

[C] 50%

[D] 55%

[E] 60%

Data for Question 6

Smith and Brown both retire at 1/1/2000. Each receives $500 at the beginning of each month as life annuities. Both are age 80 as of 1/1/2000.

P is the probability as of 1/1/2000 that Smith will receive no more than $12,000 and that Brown will receive at least as much as Smith but no more than $18,000.

Selected values:

x	q_x
80	0.0813
81	0.0885
82	0.0962
83	0.1043

Question 6

In what range is P?

[A] Less than 1.25%

[B] 1.25% but less than 2.00%

[C] 2.00% but less than 2.75%

[D] 2.75% but less than 3.50%

[E] 3.50% or more

Data for Question 7

On 1/1/2000, Smith takes a loan of $5,000. The loan agreement requires monthly payments, beginning on 1/31/2000, of 2% of Smith's monthly compensation.

Interest rate for the loan: 12% per year, compounded monthly.

Future compensation increases: 5% per year, effective each January 1.

Smith's compensation during 2000: $3,000 per month.

Question 7

In what range is the outstanding balance of the loan as of 1/1/2002?

[A] Less than $4,660

[B] $4,660 but less than $4,680

[C] $4,680 but less than $4,700

[D] $4,700 but less than $4,720

[E] $4,720 or more

Data for Question 8

$\delta = .07$

The modified duration of a 20-year bond with 7% annual coupons with a maturity and par value of $1,000 is y.

Question 8

In what range is y?

[A] Less than 10.50

[B] 10.50 but less than 10.70

[C] 10.70 but less than 10.90

[D] 10.90 but less than 11.10

[E] 11.10 or more

Data for Question 9

Type of insurance policy: Term to age 65.

Age of insured at issue: 35.

Death benefit: $100,000 payable at end of year of death.

Interest rate: 7% per year.

Selected commutation functions:

x	D_x	N_x
35	894,190	12,364,661
36	864,024	11,470,471
65	94,414	868,053
66	86,246	773,639

Question 9

In what range is the level net annual premium for the policy?

[A] Less than $415

[B] $415 but less than $425

[C] $425 but less than $435

[D] $435 but less than $445

[E] $445 or more

Data for Question 10

Terms of a 20-year annuity-certain:

Initial payment: $300 due 1/1/2000.

Payment pattern:

a) All payments are made on January 1.
b) Payments increase by $300 each year beginning 1/1/2001 through 1/1/2009.
c) Payments decrease by $200 each year beginning 1/1/2010 through 1/1/2019.

Interest rate: 7% per year, compounded annually for the first 10 years;
6% per year, compounded annually thereafter.

Question 10

In what range is the present value of the annuity as of January 1, 2000?

[A] Less than $18,600

[B] $18,600 but less than $18,800

[C] $18,800 but less than $19,000

[D] $19,000 but less than $19,200

[E] $19,200 or more

Data for Question 11

Selected values from a two-decrement table:

$q'^{(d)}_{55} = 0.009033$

$q'^{(w)}_{55} = 0.040747$

$l^{(T)}_{55} = 7120.0$

Assumptions: Deaths and withdrawals are uniformly distributed in each year of age in their respective associated single decrement tables.

Question 11

In what range is $d^{(w)}_{55}$?

[A] Less than 288.0

[B] 288.0 but less than 290.0

[C] 290.0 but less than 292.0

[D] 292.0 but less than 294.0

[E] 294.0 or more

Data for Question 12

A city is analyzing the terms of two five-year contracts for park maintenance, with each contract commencing 1/1/2000.

Terms of Contract A:

> $20,000 per month in the first year, payments at the end of each month. In each subsequent year, the monthly charge will be 2.75% higher than in the prior year.

Terms of Contract B:

> $10,000 on the 15th of each month and $10,000 at the end of each month in the first year. In each subsequent year, the semi-monthly charge will be 2.50% higher than in the prior year. In addition, a one-time payment is to be made on 1/1/2000 of $X.

Interest rate: 6% per year, compounded semi-monthly.

Question 12

In what range is $X so that the present values of the contracts are the same on 1/1/2000?

[A] Less than $1,670

[B] $1,670 but less than $4,688

[C] $4,688 but less than $7,706

[D] $7,706 but less than $10,724

[E] $10,724 or more

Data for Question 13

Date of a loan: 1/1/1990.

Amount of loan: $100,000.

Interest rate: 12% per year, compounded monthly.

Term of loan: 360 level monthly repayments.

First repayment date: 2/1/1990.

Immediately after making the 120th repayment, the borrower decides to add $Q to each monthly repayment so that the loan will be repaid after having made a total of 160 monthly repayments.

Question 13

In what range is $Q?

[A] Less than $700

[B] $700 but less than $1,250

[C] $1,250 but less than $1,800

[D] $1,800 but less than $2,350

[E] $2,350 or more

Data for Question 14

Type of insurance policy: 5-year increasing term insurance.

Initial death benefit: $10,000.

Annual increase in death benefit: 11%.

Interest rate: 7.00% per year, compounded annually.

Age at purchase: 75.

The death benefit is payable at the end of the year of death.

Selected values:

$$\frac{d_x}{d_{x+1}} = 1.11, \text{ for all x.}$$

$$\frac{d_{75}}{l_{75}} = 0.10$$

Question 14

In what range is the net single premium at age 75?

[A] Less than $4,000

[B] $4,000 but less than $4,200

[C] $4,200 but less than $4,400

[D] $4,400 but less than $4,600

[E] $4,600 or more

Data for Question 15

All of the 10,000 participants in the XYZ Company Pension Plan are subject to only the following two decrements:

Central rate of decrement from death: 0.02

Central rate of decrement from withdrawal: 0.10

There are no new entrants into the Plan and all participants are the same age.

Assumption: Total decrements at each age are uniformly distributed over the year of age.

Question 15

In what range is the expected number of participants in the Plan after one year?

[A] Less than 8,850

[B] 8,850 but less than 8,860

[C] 8,860 but less than 8,870

[D] 8,870 but less than 8,880

[E] 8,880 or more

Data for Question 16

For a certain population the survival function is:

$$s(x) = \frac{\sqrt{100-x}}{10}, \text{ for } 0 \le x \le 100$$

Question 16

In what range is the probability that a member of that population age 19 will not die between ages 51 and 64?

[A] Less than 0.8770

[B] 0.8770 but less than 0.8870

[C] 0.8870 but less than 0.8970

[D] 0.8970 but less than 0.9070

[E] 0.9070 or more

Data for Question 17

Market value of a pension fund:

Date	Value
1/1/2000	$50,000
3/31/2000	60,000
6/30/2000	45,000
9/30/2000	40,000
12/31/2000	65,000

Contributions and benefit payments:

Date	Contributions	Benefit Payments
4/1/2000	$ 0	$P
7/1/2000	17,000	P
10/1/2000	55,000	P

Dollar-weighted rate of return for 2000 using simple interest: 7.00%.

Question 17

In what range is the time-weighted rate of return for 2000?

[A] Less than 6.00%

[B] 6.00% but less than 8.00%

[C] 8.00% but less than 10.00%

[D] 10.00% but less than 12.00%

[E] 12.00% or more

Data for Question 18

An individual age 55 is purchasing a life insurance policy with the following death benefits payable at the end of the year of death.:

Age Last Birthday at Death	Death Benefit
55	$10,000
56	7,000
57	5,000
58 or greater	0

Selected values:

$e_{55} = 22.00$
$e_{56} = 21.20$
$e_{57} = 20.40$
$e_{58} = 19.60$

Interest rate: 7% per year, compounded annually.

Question 18

In what range is the benefit single premium for the life insurance policy?

[A] Less than $185.00

[B] $185.00 but less than $195.00

[C] $195.00 but less than $205.00

[D] $205.00 but less than $215.00

[E] $215.00 or more

Data for Question 19

A club has maintained a stationary population of 5,000 members during the last 30 years. A person can join only at age 30 and must resign at age 60 if still a member. There are 100 total terminations per year before age 60. The average age at termination is 35.

Question 19

In what range was the number of new members each year during the last 30 years?

[A] Less than 160

[B] 160 but less than 210

[C] 210 but less than 260

[D] 260 but less than 310

[E] 310 or more

Data for Question 20

Data from a two-decrement table:

$$l_x^{(T)} = \frac{100 - x}{e^x}$$

$d_x^{(1)} = e^{-x} - e^{-x-1}$, for all x.

Question 20

In what range is $\mu_{40}^{(1)}$?

[A] Less than -0.75%

[B] -0.75% but less than 0.25%

[C] 0.25% but less than 1.25%

[D] 1.25% but less than 2.25%

[E] 2.25% or more

Data for Question 21

A $10,000 whole life insurance policy with the death benefit payable at the end of the year of death is issued to an insured age 60.

Policy expenses:

$5 per $1,000 of death benefit, payable at the end of the year of death.
75% of first gross premium.
20% of second through tenth gross premium.
5% of eleventh and later gross premiums.
$10 at the beginning of each year.

Premiums are payable annually at the beginning of each year.

Selected commutation functions:

x	N_x
60	352,087
61	315,378
70	103,886

Interest rate: 8% per year, compounded annually.

Question 21

In what range is the gross annual premium?

[A] Less than $382

[B] $382 but less than $414

[C] $414 but less than $446

[D] $446 but less than $478

[E] $478 or more

Data for Question 22

Date of a loan: 1/1/2000.

Amount of loan: $X.

Date of first repayment: 12/31/2000.

Frequency of repayments: Annually.

Number of repayments: 10.

Amount of each repayment: $1,000.

Method of repayment:

One-half of the loan is repaid by the amortization method using an interest rate of 7% per annum compounded annually.

The other half is repaid by the sinking fund method where the lender receives 7% per annum, compounded annually, on this portion of the loan, and the sinking fund accumulates at 6% per annum, compounded annually.

Question 22

In what range is $X?

[A] Less than $6,900

[B] $6,900 but less than $7,000

[C] $7,000 but less than $7,100

[D] $7,100 but less than $7,200

[E] $7,200 or more

Data for Question 23

Normal form of retirement benefit: Life annuity.
Monthly retirement benefit: $1,500.
Date of first payment: 1/1/2000.

Actuarial assumptions:
Interest: 7% per year, compounded annually.
Mortality: A unisex mortality table to which Gompertz's law applies.
Gompertz adjustment for a 5-year age difference: 3 years.

Participant and spouse data:
Participant's date of birth: 1/1/1935.
Spouse's date of birth: 1/1/1940.

Actuarial equivalent monthly benefit under optional form: $P.

Actuarial equivalent optional form of payment: Joint & 75% contingent annuity, with 120 months certain where the participant receives $P and in the event of his death his survivor receives $0.75P. In no event will fewer than 120 payments of $P be made.

Selected commutation functions:

x	D_x	$N_x^{(12)}$	x	D_x	$N_x^{(12)}$
60	147,589	1,448,584	70	59,915	455,486
63	114,940	1,054,865	73	43,574	300,400
65	96,496	842,966	75	34,602	222,026
68	73,034	588,741	78	23,614	134,863

Selected annuity value: $\ddot{a}_{\overline{10}|}^{(12)} = 7.28714$

Question 23

In what range is $P?

[A] Less than $850

[B] $850 but less than $1,000

[C] $1,000 but less than $1,150

[D] $1,150 but less than $1,300

[E] $1,300 or more

Data for Question 24

Amount of a loan: $250,000.

Frequency of repayments: Quarterly, at the end of each quarter.

Number of repayments: 100.

Interest Rate: 8% per year, compounded continuously.

Question 24

In which repayment does the principal component first exceed the interest component?

[A] 64th

[B] 65th

[C] 66th

[D] 67th

[E] 68th

Data for Question 25

Selected annuity values:

$a_{65} = 9.70$ $a_{50} = 12.56$ $a_{20} = 14.79$

$a_{65:20} = 9.68$ $a_{65:50} = 9.48$ $a_{20:50} = 12.51$

$a_{65:20:50} = 9.44$

Question 25

In what range is $a_{65|\overline{50:20}}$?

[A] Less than 5.0

[B] 5.0 but less than 6.0

[C] 6.0 but less than 7.0

[D] 7.0 but less than 8.0

[E] 8.0 or more

ANSWER KEY

MAY 2000 COURSE EA-1, A

1. B
2. E
3. E
4. D
5. B

6. C
7. C
8. A
9. A
10. D

11. B
12. B
13. D
14. B
15. C

16. C
17. D
18. A
19. C
20. D

21. B
22. B
23. D
24. D
25. B